The ephemeris of the Moon in framework of the numerical theory of Solar system bodies EPM, IAA RAS

Yagudina E.I., Lebedeva M.A.
Institute of Applied Astronomy RAS
St. Petersburg, Russia ,19-23 July, 2024

"ANALYTICAL METHODS OF CELESTIAL MECHANICS (AMCM 2024)"

Introduction

Laboratory of Ephemeris Astronomy IAA RAS:

routinely produces lunar and planetary ephemeris EPM;
participate in the programs for the development of LLR facilities on the Earth and in space;
provides ephemeris of the Solar System bodies to the national Astronomical Yearbook and national space programs.

History.

The modern Ephemerides of Solar System bodies are created and maintained: **DE (USA), INPOP (France), EPM (Russia, IAA RAS):**

1. JPL, USA: …, DE 430, DE 440, DE 441;
2. IMCCE, Frances: …, INPOP 19a, INPOP21a;
3.IAA RAS, Russia: EPM-ERA 2014; …EPM2019, EPM2021, EPM2022a, EPM 2023a Moon…

1969-2023 - LLR observations are used for improving of the parameters of the Moon ephemeris.

The Russian EPM-ERA ephemeris from 1989 till **2014** was based on the model of orbital-rotational motion of the planets and Moon **by G.A. Krasinsky [1]** and implemented within **the ERA7 system.** During these years the dynamic model was continuously improved and approached the accuracy of the DE ephemeris parameters.

From **2014 -** a version of the ephemeris of the EPM is based on the model of the orbital-rotational motion of the Moon which is close to that used in DE430 with some rework of the **ERA7** software system in the framework of **ERA8, (D.A.Pavlov et al.[2]**).

The results of processing new LLR observations to obtain refined parameters of the **Moon EPM2023a ephemeris** within the framework of **the modernized ERA8 system** are considered.
To obtain and refine the parameters of ephemeris of **the Moon EPM2023a 33602 observations of LLR (normal points-n.p., 1985-new LLR observations)** are used.

Dynamical model

About **100 parameters of the ephemeris of the Moon EPM2023a** were improved and compared with some parameters of the ephemerides **INPOP21a** (France) and **DE440** (USA).

The values of individual parameters in different ephemerides are generally close. In the some cases discrepancies require a careful review of the list of parameters. The new results of the new ephemeris parameters of the Moon every 1-2 years are continuously analyzed.

Model

The Moon is modeled as **an elastic body with a rotating liquid core** as in model of the Moon in DE430 model [3] with currently recommended geophysical and geodynamic models.

The modeling of the orbital-rotational motion of the Moon (as part of the model of the motion of the Solar System bodies) is constructed by joint numerical integration of the relativistic **Einstein-Infeld-Hoffman equations** by the **extended Adams method** in **the BCRS inertial system using the TDB dynamic time scale. ([2])**

In the numerical integration of the Moon, accelerations to the Moon and Earth as point masses, additional disturbances from the **largest asteroids (277) and TNOs (trans-Neptunian objects) (30), TNO rings of asteroids and Kuiper, Jupiter Trojans, and the Sun's compression** are taken into account**.**

All changing of the model are taking into account in the processing of LLR observations and receiving new parameters.

The addition of new observations made it possible to refine the parameters and once again demonstrated the efficiency of using LLR observations for this purpose.

Normal points, n. p. - (from **decimeters at the beginning of 1970s to 2-5 mm at** Apache station (green-532 nm) and at GRASSE-ir (infrared-1064 nm) at 2023.

33602 LLR (N.P). 646 LLR observations were thrown out (1970-2023).

Dynamical model

Relativistic equations of motion of the planets , Sun, **Moon** in non-rotating barycentric coordinate system BCRS Solar system, are as follows:

$$\ddot{r}_{i_{point\,mass}} = \sum_{j\neq i}\frac{\mu_j}{r_{ij}^3}\left(r_j - r_i\right)\{1 - \frac{2(\beta+\gamma)}{c^2}\sum_{k\neq i}\frac{\mu_k}{r_{ik}} - \frac{2\beta-1}{c^2}\sum_{k\neq j}\frac{\mu_k}{r_{jk}} + \gamma\left(\frac{|\dot{r}_i|}{c}\right)^2 + (1+\gamma)\left(\frac{|\dot{r}_i|}{c}\right)^2 - \frac{2(1+\gamma)}{c^2}\dot{r}_i\,\dot{r}_j$$

$$- \frac{3}{2c^2}\left[\frac{(r_i - r_j)\cdot\dot{r}_j}{r_{ij}}\right]^2 + \frac{\ddot{r}_j}{2c^2}(r_j - r_i)\} + \frac{1}{c^2}\sum_{j\neq i}\frac{\mu_j}{r_{ij}^3}\{[r_i - r_j]\cdot[(2+2\gamma)\,\dot{r}_i - (1+2\gamma)\dot{r}_j]\}(\dot{r}_i - \dot{r}_j)$$

$$+ \frac{(3+4\gamma)}{2c^2}\sum_{j\neq i}\frac{\mu_j\,\ddot{r}_j}{r_{ij}}\;;$$

where are coordinates, velocities and acceleration of the *i*-th body with respect to the barycenter of the Solar system are given.

It is also necessary to add terms containing the effect of the compression of the Sun:

$$C = 3J_2\mu_s\frac{R^2}{r_{iS}^4}\left\{\left[\frac{5}{2}\left(\frac{r_i - r_s}{r_{is}}\cdot p\right)^2 - \frac{1}{2}\right]\frac{r_i - r_s}{r_{is}} - \left(\frac{r_i - r_s}{r_{is}}\cdot p\right)p\right\}$$

as well as terms containing the Lense-Thirring acceleration

$$D = \frac{2}{c^2}GS_{sun}\frac{1}{r_{iS}^3}M_{sun}\left(\dot{r_{iS}} \times z + 3\frac{z\cdot r_{iS}}{r^2}r_{iS}\times\dot{r_{iS}}\right).$$

Dynamical model

Model of the rotational motion of the Moon:

the selenocentric coordinate system position associated with the rotating the **Moon** relative to the inertial system is described by three **Euler angles**, where - **θ - is the nutation angle, ϕ - is the precession angle, and ψ - is the proper rotation angle.**

These angles (and their time derivatives) are the main values characterizing **the libration of the Moon** and participate in the numerical integration along with the position of the center **of the Moon**. All other indicators of the rotation of the Moon are calculated from **the Euler angles and their derivatives. The** matrix of the rotation of the Moon:

$$\mathbf{R}_{\mathbf{L2C}}(\mathbf{t}) = \mathbf{R}_{\mathbf{z}}\big(\boldsymbol{\phi}(\mathbf{t})\big)\mathbf{R}_{\mathbf{x}}\big(\boldsymbol{\theta}(\mathbf{t})\big)\mathbf{R}_{\mathbf{z}}\big(\boldsymbol{\psi}(\mathbf{t})\big),$$

$$\omega_x = \dot{\phi}\sin\theta\sin\psi + \dot{\theta}\cos\theta$$
$$\omega_y = \dot{\phi}\sin\theta\cos\psi - \dot{\theta}\sin\theta$$
$$\omega_z = \dot{\phi}\cos\theta + \dot{\psi},$$

Where $\boldsymbol{\omega}(\mathbf{t})$ -angular velocity of the lunar crust in the lunar coordinate system. The movement of the crust depends on ω and are in equation:

$$\ddot{\phi} = (\dot{\omega}_x \sin\psi + \dot{\omega}_y \cos\psi + \dot{\theta}(\dot{\psi} - \dot{\phi}\cos\theta))/\sin\theta$$
$$\ddot{\theta} = \dot{\omega}_x \cos\psi - \dot{\omega}_y \sin\psi - \dot{\varphi}\dot{\psi}\sin\theta$$
$$\ddot{\psi} = \dot{\omega}_z - \ddot{\varphi}\cos\theta + \dot{\varphi}\dot{\theta}\sin\theta,$$

$\boldsymbol{\omega}$ -depends on the applied torque N(t)……

A detailed description of the model of the rotational motion of the Moon taking into account all the disturbing factors influencing the rotation of the Moon (angular velocity of the Moon, the total torque acting on the lunar crust, and other factors ...) is given **in [3] and is** taken into account in detail within the framework of the **ERA8 system**

54-years of history of using LLR observations

Processing of LLR (Normal points, n. p.)

shows the efficiency of the method**:**
from decimeters in the early 1970s to 2-5 mm at Apache station now (green-532 nm) and (infrared 1064 nm-CERGA-ir).

Total 33602 observations of LLR (normal points - n.p., 1985-new LLR observations). Of these, 646 were discarded (1970-2023).

Number of LLR observations on reflectors at Table 1:
Apollo11 : 4204 (84)
Apollo14 : 3926 (81)
Apollo15 : 20557 (442)
Lunokhod-1 : 1930 (16)
Lunokhod-2 : 2339 (23)

All observations up to 2016 were available on the website:
http://polac.obspm.fr/lldatae.html;

Now all observations are posted on the website:
ftp://ccdisa.gcfc.nasa.gov/pub/slr/data/npt_crd

54 years LLR observations

Station	Years	Number of normal points
McDonald (USA)	1969-1985	3604 –
Nauchniy, Crimea, USSR	1982-1984	25 –
MLRS1, USA	1983-1988	631 –
MLRS2, USA	1988-2013	3669 –
Haleakala, USA	1984-1990	770 –
CERGA Ruby, France	1984-1996	1112 –
CERGA Yag, France	1987-2005	8316 –
CERGA MeO, France	2009- 2021	2097 –
CERGA IR, France	2015- 2023	8479 (1632)
Matera, Italy	2003- 2023	460 (**13**)
Apache Point, USA	2006- 2023	4126 (**225**)
Wettzell, Germany	2018- 2023	329 (**115**)
Total	**1969-2023**	**33602 (1985)**

EPM2023a Moon Ephemeris Parameters

The results of observation processing were obtained using **the ERA-8** software package[3], which is a Revision of the previous version, with new additions for **the epoch JD 2446000.5**;

Parameters to be specified: (about 100)

- initial coordinates and velocity components of the Moon for a given epoch;
- **Euler angles and their rate** of change for a given epoch;
- initial angular velocity of rotation of the Earth's core;
- lunar tide lag;
- The sum of the Earth-Moon masses;
- Compression ratio of the lunar core **fc**;
- coefficient of friction between the core and the crust **Kv**/**C**;
- number of Love Moon **h2**;
- dynamic parameters **β=(C-A)/B, γ=(B-A)/C;**
- tidal delay **τ;**
- Moon potential parameters;
- position of five lunar reflectors **A11, A14, A15, L1, L2**;
- observation station positions: **Haleakala/McDonald/MLRS1/MLRS2; Cerga, Apache(Apollo), Matera, Wettzell;**
- **three additional dissipative tidal terms** for the Moon;

28 shifts (biases) are non- modeled shifts in observed quantities.

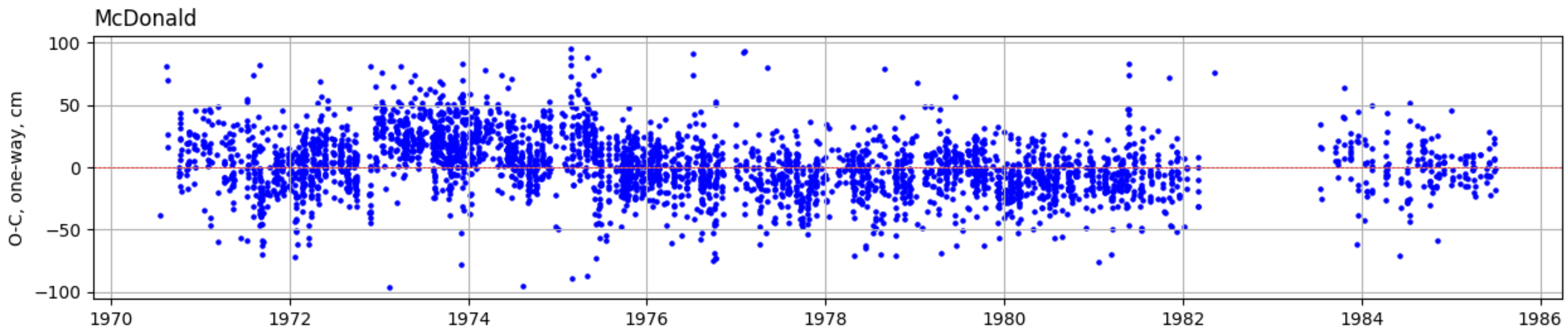
Comparison of parameters of different ephemerides

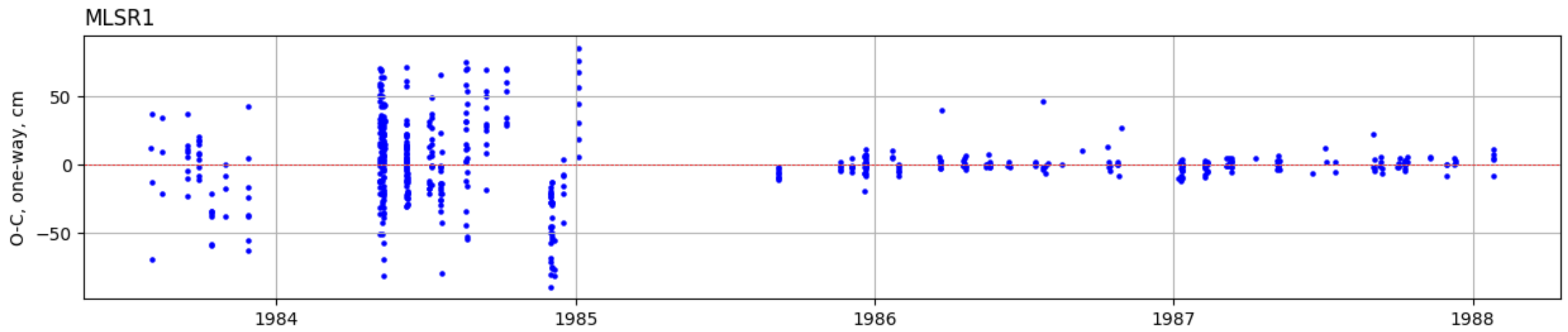
	Parameters	EPM2019	EPM2021a	EPM2023a
1	Moon X	−137136473.79 ± 0.035 m	-137136473.60 ± 0.033 m	-137136473.613 ± 0.032m
2	Moon Y	−311514604.073 ± 0.039 m	-311514604.030 ± 0.036 m	-311514603.851 ± 0.036m
3	Moon Z	−141738600.463 ± 0.065 m	−141738600.845 ± 0.059 m	-141738601.034 ± 0.059m
4	Moon v_x	962372276.463 ± 0.08 μm/sec	962372276.746 ± 0.07 μm/sec	962372276.542 ± 0.07μm/sec
5	Moon v_y	−375608190.189 ± 0.09 μm/sec	-375608189.345 ± 0.01 μm/sec	-375608189.444 ± 0.08μm/sec
6	Moon v_z	−268439309.871 ± 0.04 μm/sec	-268439309.751 ± 0.04 μm/sec	-268439309.883 ± 0.04μm/sec
7	ω_{cx}	$(-882.7 \pm 1.8) \cdot 10^{-6}$ rad/day	$(-887.5 \pm 1.6) \cdot 10^{-6}$ rad/day	$(-887.05 \pm 1.6) \cdot 10^{-6}$ rad/day
8	ω_{cy}	$(-6500 \pm 4) \cdot 10^{-6}$ rad/day	$(-6505 \pm 3.5) \cdot 10^{-6}$ rad/day	$(-6496.7 \pm 3.4) \cdot 10^{-6}$ rad/day
9	ω_{cz}	$(229.82 \pm 0.03) \cdot 10^{-3}$ rad/day	$(229.94 \pm 0.01) \cdot 10^{-3}$ rad/day	$(229.82 \pm 0.01) \cdot 10^{-3}$ rad/day
10	φ	$(-5823800 \pm 1) \cdot 10^{-8}$ rad	$(-5823800 \pm 1) \cdot 10^{-8}$ rad	$(-5823800 \pm 1)\ 10^{-8}$ rad
11	θ	$(39511600 \pm 1) \cdot 10^{-8}$ rad	$(39511600 \pm 1) \cdot 10^{-8}$ rad	$(39511600 \pm 1) \cdot 10^{-8}$ rad

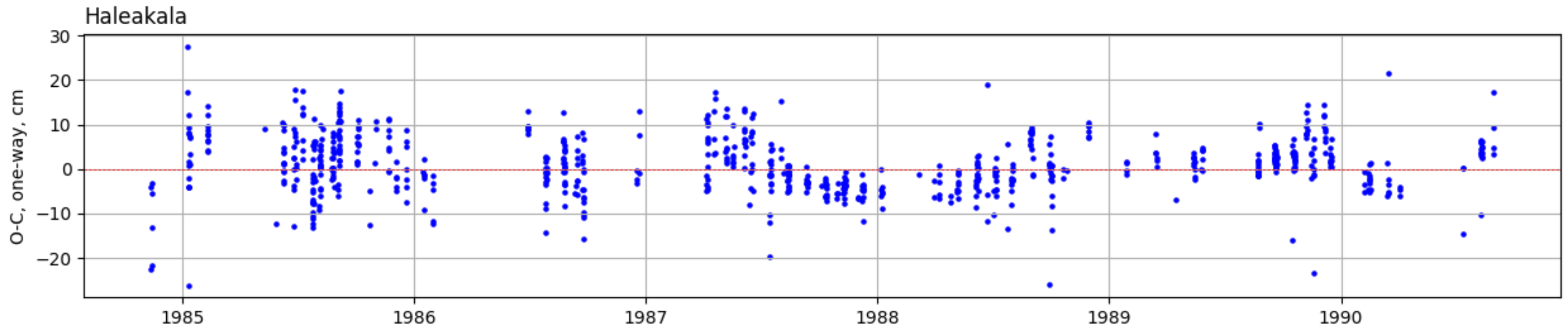
Comparison of parameters of different ephemerides

	Parameter	EPM2019	EPM2021a	EPM2023a
12	ψ	$(128918873000 \pm 3) \cdot 10^{-8}$	$(128918873000 \pm 3) \cdot 10^{-8}$	$(128918873400 \pm 2) \cdot 10^{-8}$
13	$\dot{\varphi}$	-74.533 ± 0.001“ day^{-1}	$-74.536 \pm 0.001''\mathrm{day}^{-1}$	$-74.535 \pm 0.001''\mathrm{day}^{-1}$
14	$\dot{\theta}$	$-37.0246 \pm 0.0002''\mathrm{day}^{-1}$	$-37.0253 \pm 0.0001''\mathrm{day}^{-1}$	$-37.0254 \pm 0.0001''\mathrm{day}^{-1}$
15	$\dot{\psi}$	$47501.849 \pm 0.001''\mathrm{day}^{-1}$	$47501.833 \pm 0.001''\mathrm{day}^{-1}$	$47501.851 \pm 0.001''\mathrm{day}^{-1}$
16	β	$(631020.6 \pm 0.3) \cdot 10^{-9}$	$(631022.0 \pm 0.3) \cdot 10^{-9}$	$(631023.0 \pm 0.3) \cdot 10^{-9}$
17	γ	$(227738.2 \pm 0.3) \cdot 10^{-9}$	$(227738.6 \pm 0.3) \cdot 10^{-9}$	$(227737.9 \pm 0.3) \cdot 10^{-9}$
18	τ	0.096 ± 0.001 day	0.094 ± 0.001 day	0.096 ± 0.001 day
19	h_2	0.0425 ± 0.0004	0.0427 ± 0.0004	0.0447 ± 0.0003
20	$M_E + M_M$	$403503.2364 \pm 0.0001\ \mathrm{km^3 s^2}$	$403503.2362 \pm 0.0001\ \mathrm{km^3 s^2}$	$403503.2363 \pm 0.0001\ \mathrm{km^3 s^2}$
21	k_v / C_T	$(15.9 \pm 0.1) \cdot 10^{-9}\ \mathrm{day}^{-1}$	$(16.4 \pm 0.1) \cdot 10^{-9}\ \mathrm{day}^{-1}$	$(16.1 \pm 0.1) \cdot 10^{-9}\ \mathrm{day}^{-1}$
22	f_c	$(0.260 \pm 0.002) \cdot 10^{-3}$	$(0.254 \pm 0.002) \cdot 10^{-3}$	$(0.250 \pm 0.002) \cdot 10^{-3}$

Results of processing observations by stations and years







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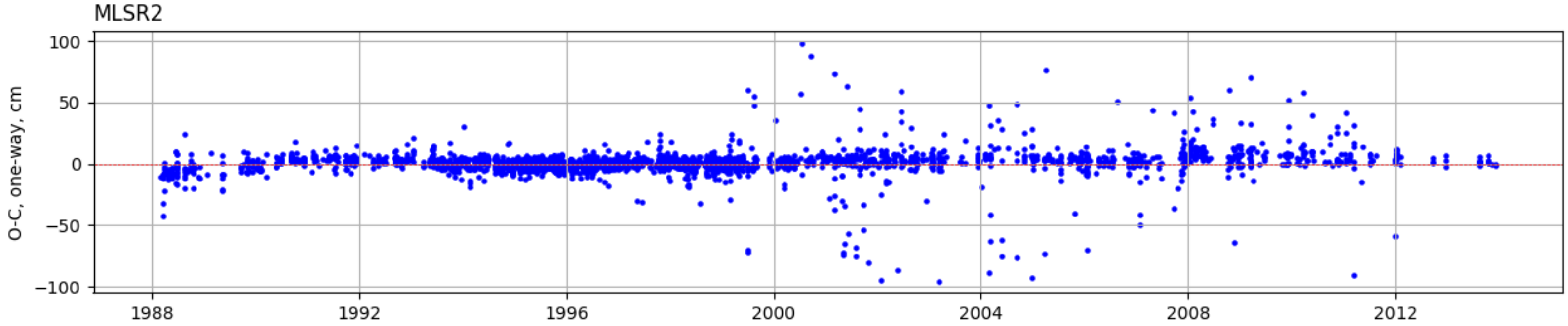
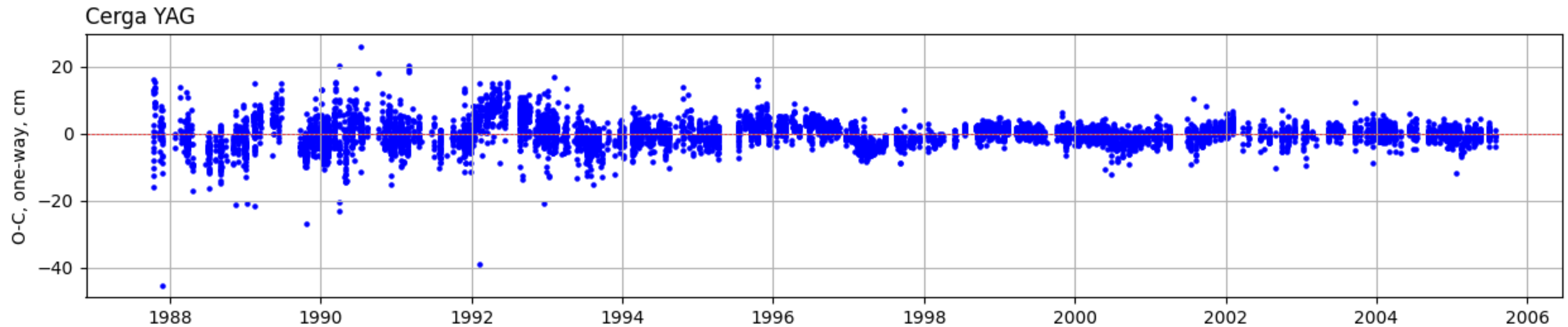
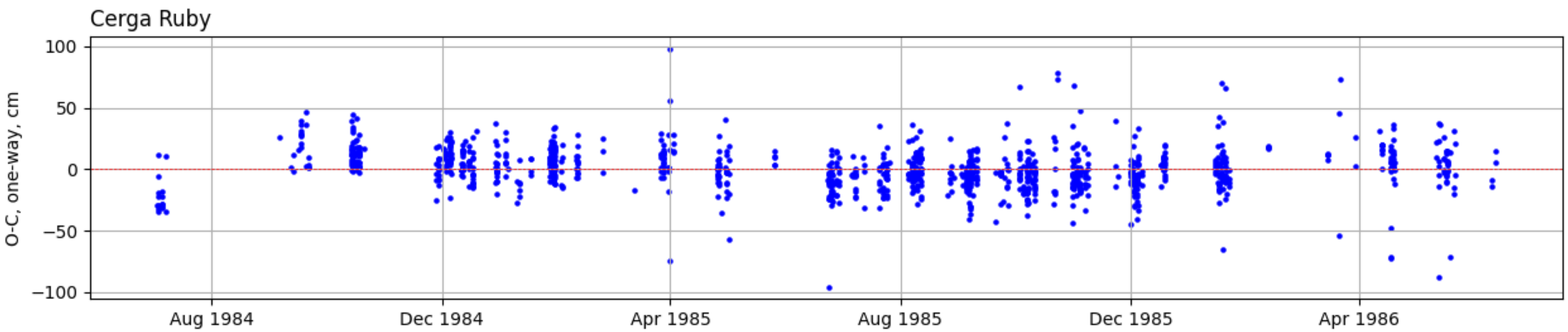
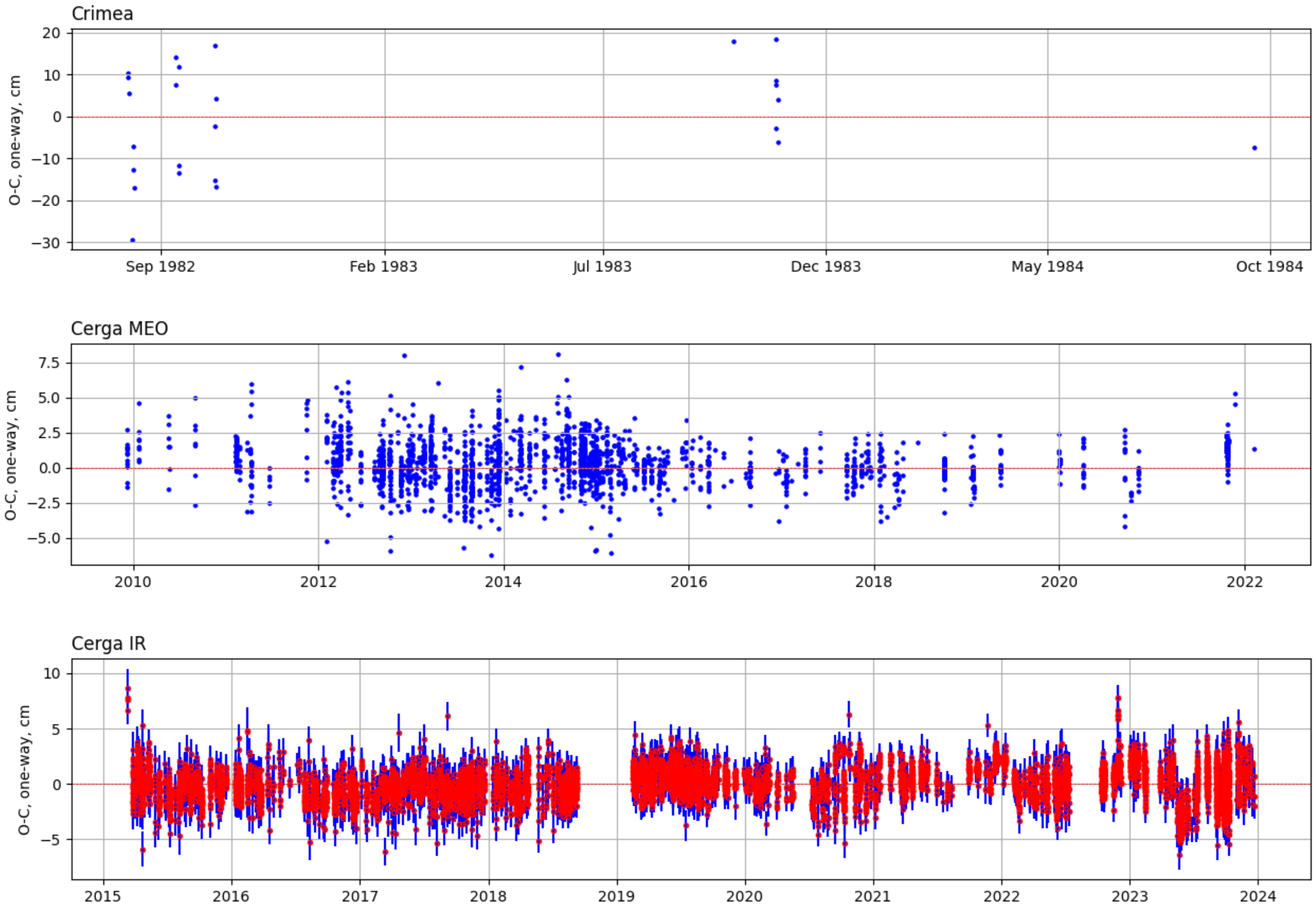




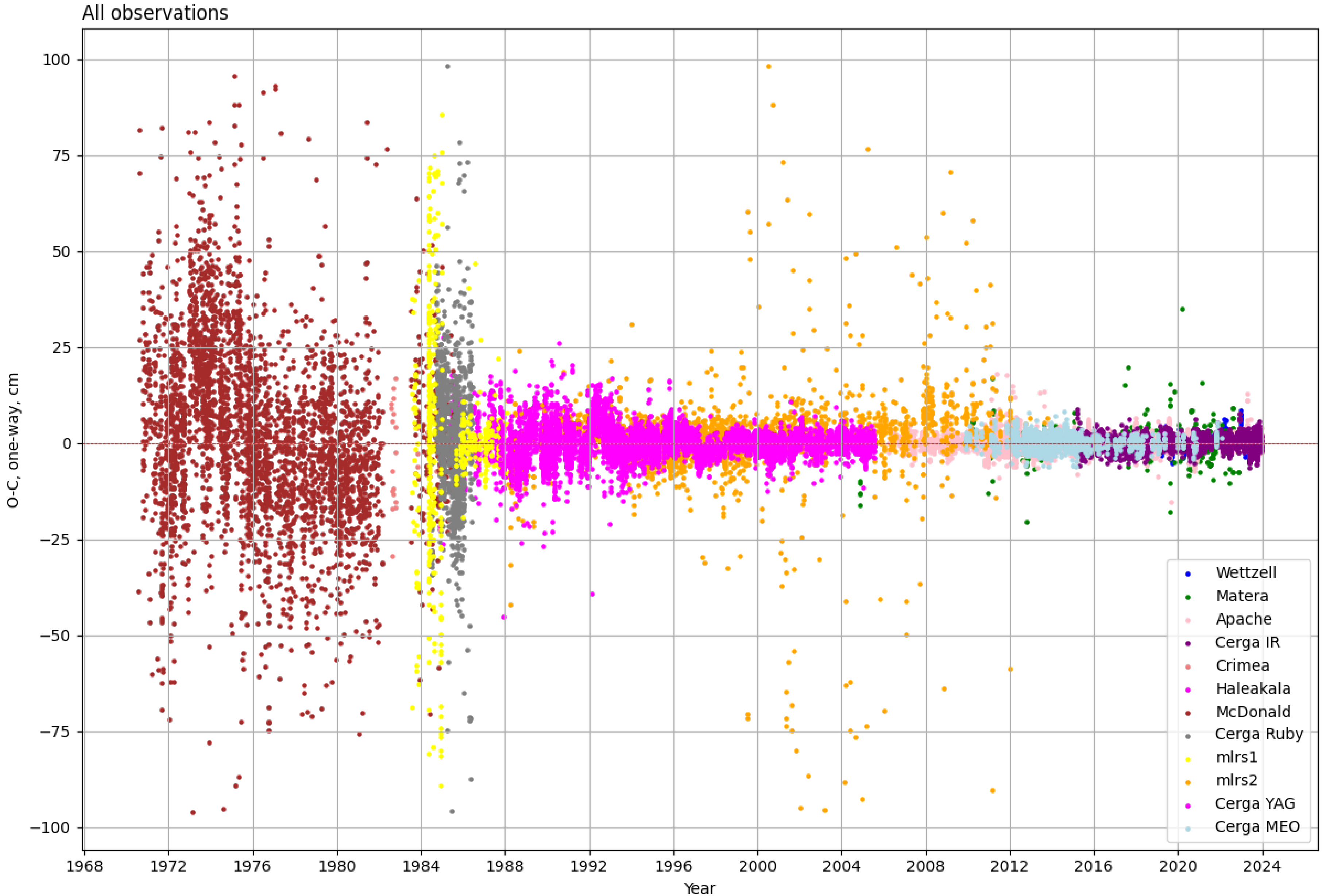


Results of processing observations by stations and years



Results of processing observations by stations and years

The results of the presentation of the processing of observations by station and year (all LLR 1970-2022)



Results of comparison the coordinates reflectors on the Moon in ephemerides EPM2023a and DE440 (in meters)

Refl	X_{DE440}	X_{EPM23a}	Y_{DE440}	Y_{EPM23a}	Z_{DE440}	Z_{EPM23a}
A11	1591967.049	1591966.865	690698.573	690699.163	21004.461	21003.740
A14	1652689.369	1652689.637	−520998.431	-520997.770	−109729.869	-109730.550
A15	1554678.104	1554678.476	98094.498	98095.267	765005.863	765005.253
L1	1114291.452	1114292.318	−781299.273	-781298.603	1076059.049	1076058.727
L2	1339363.598	1339363.663	801870.995	801871.636	756359.260	756358.713

Results of comparison of some parameters of INPOP2021a and EPM2023a

Parameter	INPOP2021a	EPM2023a
f_c-core compression ratio	2.8E-04	(2.505 ± 0.018)E-04
C_{32} −moon potential – parameter	4.84501E-06	(4.93236 ± 0.00044)E-06
h_2 Moon – Love number	**4.23E-02**	**(4.47 ± 0.03)E-02**
k_v/C_T -coefficient of friction between the core the crust	1.62E-08	(1.61 ± 0.01)E-08
τ_m - lunar tidal delay (days)	9.6E-02	(9.6 ± 0.1)E-02
Rotational delays of the earth tides $\tau_{R1\ days}$, $\tau_{R2\ days}$	8.02E-03 2.82E-03	(7.664 ± 0.019)E-03 (2.859 ± 0.002)E-03

54 years of history of using LLR observations Tabl.4 (wrms residual errors in one direction)

Station	EPM 2022a				EPM 2023a			
	Years	Normal Points.	Disc. n.p.	wrms (cm)	Years	Normal Points.	Disc. n.p.	wrms (cm)
McDonald, USA	1970-1985	3588	34	**21.5**	1970-1985	3588	34	**21.7**
MLRS1, USA	1983-1988	631	46	**8.9**	1983-1988	631	46	**8.8**
MLRS2, USA	1988-2015	3669	389	**3.6**	1988-2015	3669	389	**3.6**
Haleakala, USA	1984-1990	770	23	**5.2**	1984-1990	770	23	**5.3**
Cerga, Ruby (Fr.)	1984-1986	1112	3	**16.7**	1984-1986	1112	3	**16.7**
Cerga, (Fr.)YAG	1987-2005	8316	39	**2.3**	1987-2005	8316	39	**2.3**
Cerga, (gr.) (Fr.) MeO	2009-2022	2097	0	**1.5**	2009-2022	2097	0	**1.5**
Cerga , IR	2015-2022	6847	7	**1.2**	2015-2023	8479	4	**1.1**
Apache	2006-2022	3901	78	**1.4**	2006-2023	4126	77	**1.5**
Matera	2003-2022	421	2	**3.2**	2003-2023	460	28	**3.4**
Wettzell	2018-2022	212	0	**1.5**	2018-2023	329	3	**1.6**

Conclusions

At present, all 3 ephemerides of the Moon (DE, INPOP, EPM) have approximately the same accuracy. The accuracy of modern LLR observations is still higher than the accuracy of the parameters determined from the ephemerides:
2-4 mm at Apache station (532 nm);
at the level or even slightly better than infrared observations at Cerga station (1064 nm).
2-5 cm – from the ephemerides.

Further steps to refine the lunar ephemeris
I. Possibilities for improving the accuracy of the EPM ephemeris parameters:
Refining and checking new observations, changing and refining individual parameters.
It is necessary to continue to investigate the cause of the outlier in LLR observations at individual stations and introduce **biases.**

II. In previous works (Vasiliev M.V., Yagudina E.I.) it was shown on the basis of mathematical modeling that there are several ways to improve the parameters of the Moon's ephemeris:
1. adding a LLR observation station [7]; (up to 12%)
2. involving radar observations of the Moon, [8]; (from 20% to 60%).
3. VLBI observations [9], [10]. (at the LLR level accuracy +/-%)

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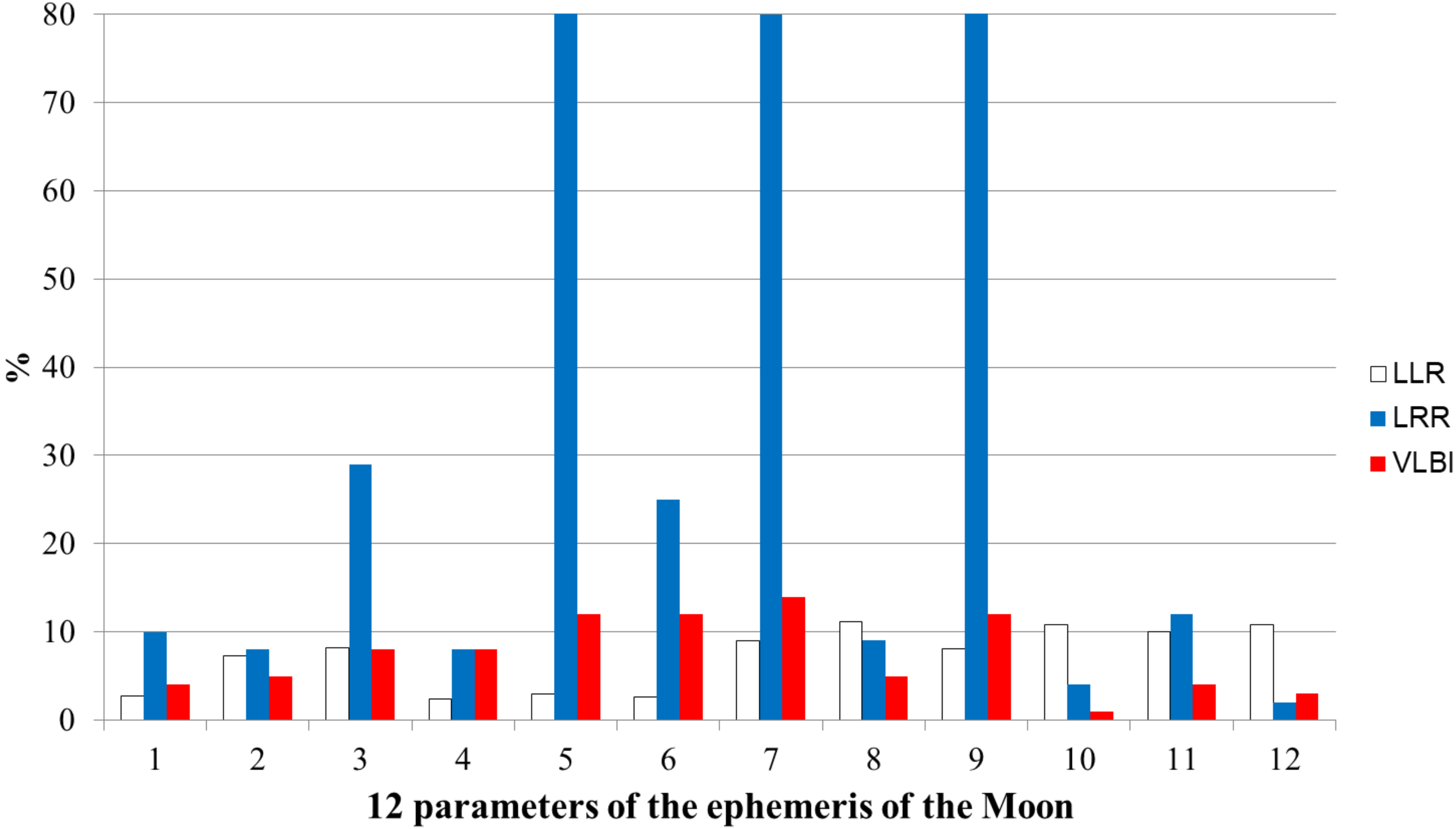
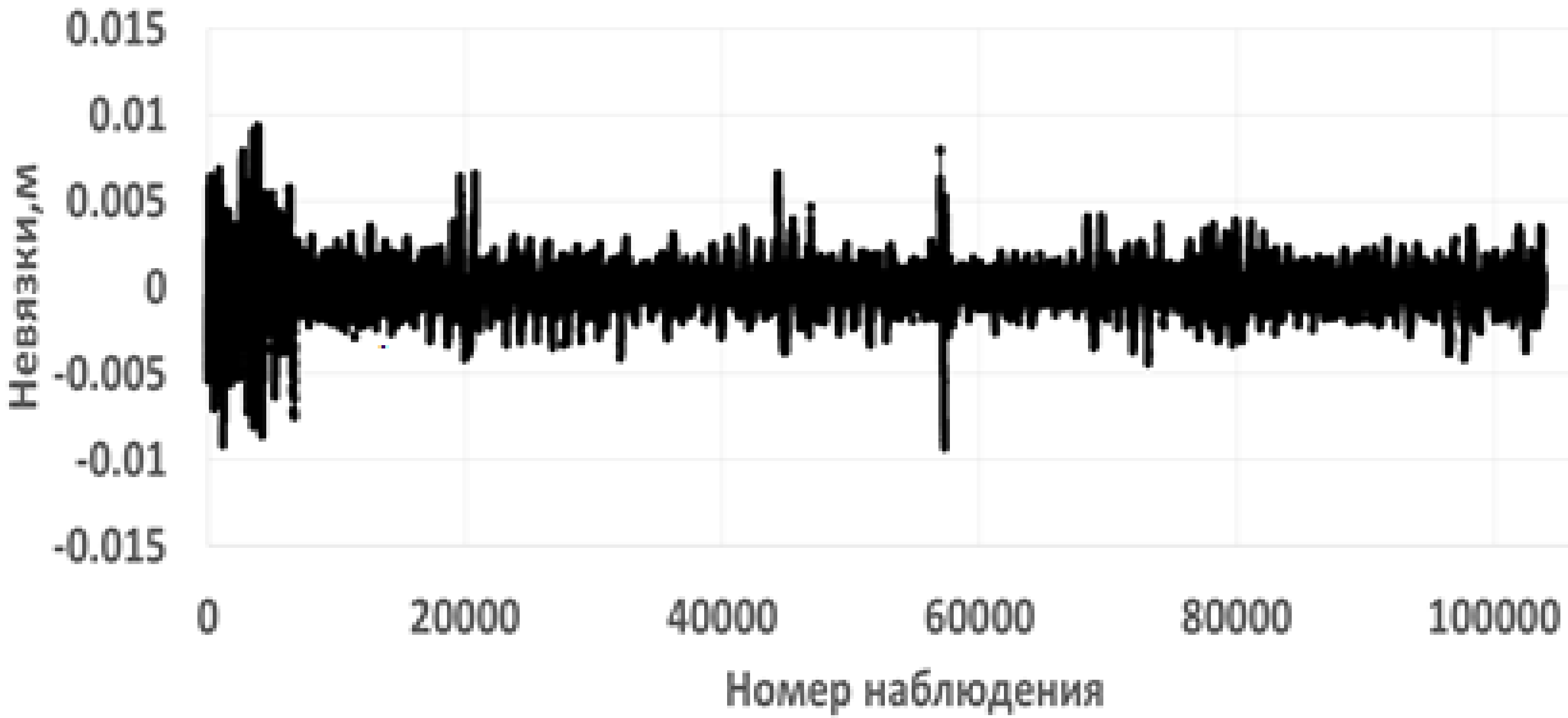
Thank you very much for your attention !

Expected impact of new LLR stations and LRR,VLBI observations on the lunar ephemeris accuracy



“Preliminary results of processing phase radar observations of lunar landers” Vasiliev M.V., Yagudina E.I., Sharkov V.S., Bondarenko Yu.S., Marshalov D.A. KVNO-2019

“Предварительные результаты обработки фазовых радиолокационных наблюдений лунных посадочных аппаратов” Васильев М.В., Ягудина Э.И., Шарков В.С., Бондаренко Ю.С., Маршалов Д.А. КВНО-2019



Остаточные невязки ФРН-наблюдений

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Russia, IAA RAS Прежде чем переходить к изложению своего сообщения, хотелось бы от своего имени и от других сотрудников нашего Института (IAA RAS) выразить благодарность и уважение Орг комитету и Институту Эйлера за посвящение конференции памяти Константина Холшевникова- известного ученого, прекрасного, педагога и удивительную личность.

Все, кто либо учились у него, либо контактировали по работе, либо просто общались, не могут не согласиться с тем, что КХ был и остается в их памяти интереснейшим человеком, с удивительным чувством юмора и умением, не обижая собеседника, выразить свое истинное мнение.

В ИПА РАН КХ, где он частенько бывал , помимо представления интересных докладов, он мог своим лишь одним вопросом изменить направление дискуссии, вызвать восхищение, спровоцировать спор или открыть новое направление для изучения данной темы.

Спасибо Организаторам конференции за возможность напомнить о нем Всем, кто учился, работал или просто общался с КХ. Все мы будем помнить КХ, а его имя в науке будет всегда. Светлая память и благодарность от нас всех!